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String Embeddings of the Pentagon

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ABSTRACT: The Pentagon Model is an explicit supersymmetric extension of the Standard Model, which involves a new strongly-interacting $SU(5)$ gauge theory at TeV-scale energies. We discuss embeddings of the Pentagon Model into string theory, specifically $\mathcal{N} = 1$ supersymmetric type IIa intersecting D-brane models, M-theory compactifications of G_2 holonomy, and heterotic orbifold constructions.

Contents

1. Introduction

The string model-building program has a number of goals. First, if completely realistic models are found, this would provide a proof that string theory may be a unified theory of all particles and interactions. Further, the study of the surviving low-energy spectra of various string models might lead to the identification of general patterns (such as symmetries or exotic particle content) present in a large class of realistic vacua. Additionally, it might lead to new ideas for addressing problems such as CP violation, fermion mass mixings, or even dark matter and dark energy. Perhaps most significant is the hope that the discipline will lead to experimentally testable predictions. The last of these is especially provocative at a time when we find ourselves on the verge of a plethora of new data from the LHC.

The Pentagon Model of TeV physics successfully addresses a number of low energy phenomenological issues, and we would therefore like to find it as an effective field theory of a string construction. Such a search is the subject of this paper.

Though previous search [1] has produced promising results towards embedding the Pentagon into a grand unified theory (GUT), the method nevertheless relies on arguments involving operators at the Planck scale where in reality the theory breaks down and becomes unreliable. Thus, the purpose of this paper is to explore the possibility of embedding the Pentagon Model into a string theory directly. In practice this translates into choosing one specific string theory with a given geometry and turning the crank to find the resulting particle spectrum of that theory, and comparing it with the Pentagon. There are currently five different types of string theories for which it is known how to calculate the low-energy chiral spectrum: orbifold constructions in heterotic string theory, G_2 compactifications of M-theory, intersecting D-brane models in either type IIA or IIB string theory, and F-theory models. In this chapter we will consider the first three of these approaches. Type IIB and their dual F-theory models might certainly be of interest, but are left to future work.

Though a search for the Pentagon has never before been performed, it should be noted that each of these approaches has yielded only mediocre results in previous searches for the standard model and various GUTs. While many models have been discovered which may contain the desired particle content and gauge symmetries, one must also contend with other issues such as problems with the existence of chiral exotics, symmetry breaking and Higgs fields, and finding proper $U(1)$ charges and Yukawa couplings. While extensive research has been devoted to these questions, only perhaps a handful of models have satisfactorily addressed all of these issues. As the purpose of this search is merely to establish the viability of the existence of the Pentagon, our strategy has been to search for models that are 'at least as good as state of the art'. In other words, we must begin by searching the various string theories for our desired particle content. If we were to find a massless particle spectrum corresponding to that of the Pentagon, we would then turn our attention to the phenomenological aspects of these models.

Unfortunately, we have found that the existence of the Pentagon as a low energy spectrum of these theories is impossible at worst and inconclusive at best. The difficulty seems to arise due to the requirement that we obtain both chiral and vector-like particles, as will be discussed. The paper is constructed as follows: We first briefly review the contents of the Pentagon Model in section 2. In section 3 we will consider models of $\mathcal{N} = 1$ globally supersymmetric type IIA intersecting D6-brane construction. In [10], Cvetič, Papadimitriou, and Shiu (CPS) have performed an extensive search for $\mathcal{N} = 1$ supersymmetric three-family $SU(5)$ Grand Unified Models in the type IIA con-

text, and we therefore use their work as the starting point for our search. In Section 4 we consider the G_2 lift of these models onto eleven dimensional M-theory compactifications. We find a possible candidate for the Pentagon in this context, but cannot provide a proof for its existence. Section 5 is devoted to heterotic orbifold models. It appears difficult to find a consistent model supporting an $SU(5) \times SU(5)$ gauge group with charged matter in the bifundamental representation, so we instead focus our search for the chiral spectrum of the Pyramid Model. The results of some of the more promising models are listed but we were unable to find an exact replication of the low energy model, though we were unable to rule out the possibility. In Section 6 we write some concluding remarks.

2. The Pentagon

2.1 The Original Model

The Pentagon is a supersymmetric model of TeV scale physics [2, 3], whose foundation is the Minimally Supersymmetric Standard Model (MSSM). The Pentagon model was origianly constructed to address standard issues with the MSSM, such as SUSY breaking, the μ problem, the flavor problem, CP violation, and baryon violation . In addition to an $SU(5)$ grand unified version of the MSSM, a new strongly interacting 'Pentagon' $SU(5)$ super-QCD with five flavors of pentaquarks is introduced as a hidden sector which mediates SUSY breaking through Standard Model gauge couplings. An hypothetical meta-stable $N_F = N_C = 5$ vacuum of the theory is used to employ the SUSY breaking mechanism of Intriligator, Seiberg, and Shih (ISS) to construct an effective theory for Cosmological SUSY breaking (CSB). It naturally introduces a μ term of the right order of magnitude, contains a discrete R-symmetry which eliminates all unwanted dimension 4 and 5 Baryon and Lepton violating operators, and resolves the SUSY flavor problem. Strong CP violating phases remain in the model (in addition to standard neutrino see-saw and CKM matrix phases), but these are potentially addressed with the addition of an axion.

The Lagrangian of the Pentagon model contains several pieces. The standard MSSM Lagrangian is implemented as usual: the kinetic energy terms for the matter and Higgs fields arise in the Kahler potential,

$$\mathcal{L}_1 = d^4\theta[P^*e^V P + Q^*e^V Q + L^*e^V L + (\bar{U})^*e^V \bar{U} + (\bar{D})^*e^V \bar{D} + (\bar{E})^*e^V \bar{E}$$

and the gauge superpotential produces the kinetic terms for the gauge fields and gauginos,

$$\mathcal{L}_2 = \int d^2\theta (\sum \tau_i W_\alpha^i)^2.$$

Yukawa couplings for the Standard Model fermions and a mass term for the Higgsino are contained in the superpotential,

$$\mathcal{L}_3 = \int d^2\theta \lambda_u H_u Q\bar{U} + \lambda_d H_d Q\bar{D} + \lambda_L H_d L\bar{E} + \frac{\lambda^{mn}}{M_U} L_m L_n H_u^2 + h.c.$$

In addition to the MSSM Lagrangian, the Pentagon model includes an additional superpotential for the pentaquarks (transforming as $P \sim [5, \bar{5}]$ and $\tilde{P} \sim [\bar{5}, 5]$ under the $SU(5)_P \times SU(5)_{GUT}$ gauge group) and an additional singlet field S with discrete R-charge 2:

$$\mathcal{L}_4 = \int d^2\theta P_i^A \tilde{P}_A^j (m_{ISS}\delta_j^i + g_S S Y_j^i) + g_\mu S H_u H_d + g_T S^3.$$

The scale M_U is taken to lie in the range $M_U \sim 10^{14} - 10^{15}$ GeV to successfully implement the neutrino seesaw effect. m_{ISS} is assumed to be induced by CSB in the UV sector of the theory (we will discuss CSB further in the next section),

$$m_{ISS} = \gamma \frac{\lambda^{1/4} M_P}{\Lambda_5}.$$

λ is the cosmological constant, M_P the Planck mass, and Λ_5 the confinement scale of the Pentagon gauge group. To be consistent with CSB, $\Lambda_5 \sim 1.5$ TeV. γ is an unknown constant of order one. ISS proved that for a theory of SUSY QCD with $N_C + 1 \leq N_F \leq \frac{3N_C}{2}$, the mass term $m_{ISS} \text{Tr} P\tilde{P}$ induces a meta-stable SUSY violating ground state with SUSY order parameter $F \sim m_{ISS}\Lambda_{N_C}$[1] [4]. They further argued that a similar meta-stable state might exist for a theory with $N_F = N_C$, though its properties could not be calculated analytically. The Pentagon therefore has a stationary point of its effective potential with a non-zero vacuum energy of order $m_{ISS}^2 \Lambda_5^2$. SUSY breaking is communicated to the Standard Model via two mechanisms. The dominant contribution to gaugino masses as well as the masses of the squarks and sleptons is through conventional gauge mediation. The Higgs superfields also contribute tree level masses to squarks and sleptons due to non-zero F terms.

The singlet field S is thought to be the remnant of an $SU(5)$ adjoint, transforming like the hypercharge generator of $SU(3) \times SU(2) \times U(1)$. Its coupling to the Standard Model therefore implies that the GUT $SU(5)$ is broken to $SU(3) \times SU(2) \times U(1)$. It also ties the properties of the meta-stable SUSY violating vacuum to electroweak symmetry breaking through its F-term, predicting $SU(2) \times U(1) \to U(1)_{EM}$ with $|h_u| \sim |h_d| \sim \Lambda_5$, $\tan\beta \sim 1$. Furthermore, the VEV of S can give rise to a natural μ term.

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[1]The analysis is only under analytic control if $m_{ISS} << \Lambda_{N_c}$

The SUSic $m_{ISS} \to 0$ limit of the theory admits an anomaly free R-symmetry which is identified with the discrete Z_N R-symmetry required by the rules of CSB. The SUSY degrees of freedom transform non-trivially under an R-symmetry, it follows that $N = 4$ to accommodate all terms in the superpotential. In models of CSB, the discrete R-symmetry guarantees Poincaré invariance; it also has the effect of preventing all unwanted dimension 4 and 5 baryon and lepton violating operators leading to proton decay[2]. The Z_4 also forbids various dimension 5 flavor combinations, so quark and lepton flavor changing processes arise from dimension 6 operators. Thus, similar to generic gauge mediated models, flavor changing neutral currents are suppressed below experimental limits.

R-parity preservation implies that the LSP is the gravitino. Estimates of the scale of SUSY breaking give a gravitino mass of order 5×10^{-3} eV, consistent with Big Bang Nucleosynthesis. It is far too small, however, to be a viable dark matter candidate, and it is strongly coupled enough that the NLSP will decay too rapidly to be of cosmological importance. Thus there is no conventional MSSM dark matter candidate. On the other hand, ISS show that the Pentabaryons, dimension one fields made of five pentaquarks, have a non-vanishing expectation value in the meta-stable vacuum. Pentabaryon number is therefore spontaneously broken,

$$\langle B \rangle = \Lambda_5 e^{ib/\Lambda_5},$$

and the associated Goldstone boson, the penton, is cosmologically long-lived. If the penta-baryon asymmetry produced in the early universe is sufficiently large, the penton can be the dark matter. Furthermore, the pentabaryon and baryon numbers are coupled by QCD interactions, providing a possible connection between the dark matter and the observed baryon asymmetry of the universe. We will discuss the issues of dark matter and baryogenesis further in the next section.

2.2 The Pyramid

After its invention, it was noticed that the Pentagon model may suffer from a number of troubling issues. Most importantly, $\Lambda_5 \sim 1.5$ leads to a Landau pole before gauge coupling unification. In fact, a calculation of the two loop β functions for the running of Standard Model couplings requires both $\Lambda_5, m_{ISS} > 10^3$ TeV [5]. This is inconsistent with the conditions of CSB. Another problem has to do with stellar phenomenology. The penton gains mass through a dimension 7 operator; if the scale associated with

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[2]This symmetry is explicitly broken by the ISS mass term, but the arguments of CSB lead us to believe that these operators will still be supressed when the cosmological constant is non-zero, i.e. in the SUSY broken theory.

this operator is too large, stars will produce an overabundance of pentons leading to unobserved stellar cooling [6].

The successor of the Pentagon, the Pyramid model, was constructed to address these issues [7]. The Pyramid model employs an $SU(3)^4$ gauge symmetry, each factor being represented by the vertices of a pyramid quiver diagram. Standard Model particles exist as broken multiplets running around the base of the pyramid–singlets of a new Pyramid $SU(3)_P$ gauge group, but fitting into complete multiplets of a conventional trinification GUT. In such models, a single generation of fermions comes in the representation

$$(3, 1, \bar{3}) \oplus (\bar{3}, 3, 1) \oplus (1, \bar{3}, 3)$$

under the trinification $SU_1(3) \times SU_2(3) \times SU_3(3)$. This respects a Z_3 permutation symmetry, and can be embedded precisely into the **27** of E_6. $SU_3(3)$ is identified with the color symmetry of the Standard Model, electroweak symmetry comes from an $SU(2)$ subgroup of $SU_2(3)$, and hypercharge is a linear combination of generators from both $SU_2(3)$ and $SU_1(3)$. Gauge coupling unification is guaranteed if all matter comes in complete representations of $SU(3)^3 \times Z_3$ and this symmetry is preserved by Yukawa couplings.

Analogous to pentaquarks, trianons are introduced to implement the ISS mechanism of meta-stable SUSY breaking and to mediate SUSY breaking to the MSSM. Trianons transform under both the Pyramid $SU(3)_P$ and the trinification symmetry:

$$\mathcal{T}_1 + \bar{\mathcal{T}}_1 = (3, 1, 1; \bar{3}) + (\bar{3}, 1, 1; 3)$$

$$\mathcal{T}_2 + \bar{\mathcal{T}}_2 = (1, 3, 1; \bar{3}) + (1, \bar{3}, 1; 3)$$

$$\mathcal{T}_3 + \bar{\mathcal{T}}_3 = (1, 1, 3; \bar{3}) + (1, 1, \bar{3}; 3)$$

Because they respect the Z_3 symmetry, one loop perturbative coupling unification is preserved.

The remainder of the construction of the theory is in complete parallel with the Pentagon model. The singlet field S can give rise to a μ term, and its F-terms gives a VEV to the meson fields that are responsible for electroweak symmetry breaking. A discrete R-symmetry exists as a consequence of CSB which forbids all dangerous dimension 4 and 5 operators. Gaugino and squark masses are estimated to lie in an acceptable range for phenomenology. The pyrmabaryons themselves are expected to be the prime dark matter candidate, although spontaneous breaking of pyrmabaryon number does occur in the model. The Goldstones of this broken symmetry are called the pyrmions which, in contrast to the pentons, avoid constraints from stellar cooling.

Although the majority of this paper is devoted toward developing the Pentagon model, we do address how the Pyramid model can be extended to accommodate these developments.

3. Intersecting type IIA D-branes

3.1 Brief review

Intersecting D-brane models provide a very nice geometric picture for some of the fundamental ingredients of any low energy effective field threory[3]. In particular, they provide a mechanism for generating not only gauge symmetries but also chiral fermions, where family replication is achieved by multiple topological intersection numbers of various D-branes. To be more specific, the spectrum of open strings stretched between the intersecting D-branes contains the chiral particles which are localized at the intersections. In this section we will consider specifically the construction of four dimensional $\mathcal{N} = 1$ supersymmetric type IIA orientifolds with D6-branes intersecting at angles.

Type IIA superstring theory exists in 10 space-time dimensions, six of which must be compactified to make contact with the observed world. The theory contains both closed and open strings as well as extended charged objects of higher dimension–the D-branes. Fluctuations of these objects can be described as open strings attached to the D-branes. The endpoints of the strings give Chan-Paton factors, which can be viewed as a $U(1)$ gauge field with momentum only along (and therefore confined to) the brane. By placing N D-branes on top of each other the gauge fields on the branes will transform in the adjoint representation of the gauge group $U(N)$. If these fields are carried by D6-branes, three dimensions must remain uncompactified for these fields to be free to move in four dimensional Minkowski space-time. This means that in the six dimensional transverse compact space the branes are three dimensional and wrap a three dimensional cycle. In general, two such branes will intersect at a point in the compactified space.

An open string extended between the two branes can be shown to have only one fermionic degree of freedom. Taking into account an open string with the opposite orientation between the two D6 branes, one is left with two fermionic degrees of freedom corresponding to one chiral Weyl fermion from the four dimensional point of view. In the same way, strings extended at the intersection of two stacks of branes, with N and M D6-branes per stack respectively, will give rise to a chiral fermion transforming in the bifundamental representation of $U(N) \times U(M)$. While the gauge fields are

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[3]For a review, see [8].

confined to the branes, gravity still propagates throughout the bulk. Thus, the D-branes interact gravitationally, which means that they will contribute positively to the vacuum energy. To cancel this contribution, we must introduce negative tension objects known as orientifold planes. Both the D-branes and the orientifold planes carry R-R charge, which must vanish for consistency. This gives rise to tadpole cancellation conditions, which must be satisfied along with certain supersymmetry conditions for the theory to be consistent.

The simplest compactification scheme is six dimensional toroidal compactification factorized as the product of three rectangular two-tori, $T^6 = T^2 \times T^2 \times T^2$, and to assume that the D6-branes are the products of one-cycles in each of the three two-tori. This allows us to specify the branes by wrapping numbers (n^i, m^i) along the fundamental cycles $[a^i]$ and $[b^i]$ on the ith T^2. Next we introduce the orientifold O6-plane, and allow it to wrap along each of the $[a^i]$ cycles (as well as the transverse uncompactified space). The introduction of the orientifold plane mods the theory by world-sheet parity as well as an anti-holomorphic involution, so that the 06-plane is localized at the fixed plane of the local reflection $(n^i, m^i) \rightarrow (n^i, -m^i)$. However, in this scenario, if the D6-branes do not lie entirely parallel to the 06-plane everywhere, the tension of these branes in the perpendicular directions cannot be canceled. Thus, no non-trivial globally supersymmetric consistent models can be constructed on these manifolds.

This problem can be alleviated by extending the orientifold planes into all perpendicular directions via orbifolding[9]. The simplest examples of such models are orientifolds of toroidal type IIA orbifolds $T^2 \times T^2 \times T^2/(Z_2 \times Z_2)$. Using the notation of [9, 10], the orbifold twists are $v = (1/2, -1/2, 0)$ and $w = (0, 1/2, -1/2)$, acting on the complex coordinates of the three two-tori as

$$\Theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3)$$

$$\omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3).$$

Orientifolding mods the theory by the orientifold action ΩR, where Ω is world-sheet parity and R acts as

$$R : (z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3).$$

As with the case of toroidal compactification, the action of the orientifold requires the O6-plane to lie along the three $[a_i]$ cycles. However, orbifolding creates three new classes of O6-planes, each associated with the combined action of the orientifold and the orbifold:

$$\Omega R : [\Pi_1] = [a_1][a_2][a_3], \quad \Omega R\Theta : [\Pi_2] = [b_1][b_2][a_3],$$

$$\Omega R\omega : [\Pi_3] = [a_1][b_2][b_3], \quad \Omega R\Theta\omega : [\Pi_4] = [b_1][a_2][b_3].$$

The complex structure of the tori is arbitrary but must be consistent with the orientifold projection. This admits only two choices, each torus may be rectangular (with the lattice vectors $e_1 \perp e_2$) or tilted such that $e'_1 = e_1 + e_2/2, e'_2 = e_2$. To describe both choices in a common notation, a generic one cycle can be written as $n_a^i[a_i] + l_a^i[b_i]$, with $l_a^i = m_a^i$ for a rectangular torus and $l_a^i = 2m_a^i + n_a^i$ for a tilted torus. The homology class of a three cycle is just the product of three one cycles,

$$[\Pi_a] = \prod_{i=1}^{3}(n_a^i[a_i] + 2^{-\beta^i}l_a^i[b_i])$$

where the factor $2^{-\beta^i}$ is included to account for tilted tori ($\beta_i = 1$ if the ith torus is tilted, zero otherwise). The orientifold action maps a one cycle (n_a^i, l_a^i) to its image $(n_a^i, -l_a^i)$, thus for any stack of D-branes we must also include its image

$$[\Pi'_a] = \prod_{i=1}^{3}(n_a^i[a_i] - 2^{-\beta^i}l_a^i[b_i]).$$

Finally, we define

$$[\Pi_{O6}] = 8[\Pi_1] - 2^{3-\beta_1-\beta_2}[\Pi_2] - 2^{3-\beta_2-\beta_3}[\Pi_3] - 2^{3-\beta_1-\beta_3}[\Pi_4].$$

The coefficients reflect the number of images of each O6 plane that must be included.

With these definitions we are equipped to consider the open-string spectrum of the theory. Chiral sectors are defined by the objects between which the strings in the sector are extended. Adjoint fields are given by strings with endpoints on a single brane, thus the gauge group is found in the aa sector. As mentioned, in toroidal theory a stack of N_a D6-branes gives rise to a $U(N_a)$ gauge group. In the orbifold theory, the Θ action breaks this to $U(N_a/2) \times U(N_a/2)$, and the ω action identifies these factors, leaving the gauge group $U(N_a/2)$. However, in the special case of branes coincidental with some of the O6-planes, the symmetry is enhanced to a $USp(N_a)$ gauge group. Massless strings extended between these branes will necessarily be vector-like, and so they have gained the name 'filler branes' because they can contribute an RR tadpole charge without adding to the particle spectrum.

The $ab + ba$ sector gives chiral supermultiplets in the bi-fundamental representation $(N_a/2, \overline{N_b/2})$. The multiplicity of these states is given by the topological intersection number

$$I_{ab} = [\Pi_a][\Pi_b] = 2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i)$$

with $k = \beta_1 + \beta_2 + \beta_3$. Similarly, the $ab' + b'a$ sector (the prime indicates the ΩR image) gives $I_{ab'}$ chiral fields in the representation $(N_a/2, N_b/2)$, with

$$I_{ab'} = [\Pi_a][\Pi_b'] = -2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i + n_b^i l_a^i).$$

The sign of I signifies the chirality of the particle, with a negative intersection number corresponding to a left-handed fermion.

D6-branes can also intersect with their images. Naively one might assume that strings extended from a stack a to stack a' would give particles transforming as $(N_a/2, \overline{N_a/2})$, but the orientifold projection leads to two index symmetric and antisymmetric tensor representations of $U(N_a/2)$. The intersection number between a stack and its image is given by

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k} \prod_{i=1}^{3} (n_a^i l_a^i).$$

However, massless strings will also stretch between a stack of branes and image at the orientifold planes, and so we must take into account the intersection

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3).$$

The final result for the net number of symmetric and anti-symmetric representations is found by anomaly cancellation:

$$\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6}) \quad \text{symmetric}$$

$$\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6}) \quad \text{antisymmetric}.$$

The resulting chiral spectrum is listed in table 1.

A consistent supersymmetric theory must satisfy both tadpole and supersymmetry constraints. Cancelation of RR tadpoles follows from the cancellation of D6-brane and O6-plane charge, which implies

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0.$$

To preserve supersymmetry, each D6-brane must be related to the orientifold plane by an $SU(3)$ rotation. Because the D6-branes are taken to be products of one-cycles, each cycle will lie at some angle θ_i with respect to the horizontal direction in the ith torus. The condition

$$\theta_1 + \theta_2 + \theta_3 = 0 \mod 2\pi$$

Sector	Representation	Multiplicity
$ab + ba$	$(N_a/2, \overline{N_b/2})$	$I_{ab} = 2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i - n_b^i l_a^i)$
$ab' + b'a$	$(N_a/2, N_b/2)$	$I_{ab'} = -2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i + n_b^i l_a^i).$
$aa' + aO6$	$(N_a \otimes N_a)_s$ $(N_a \otimes N_a)_a$	$\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6})$ $\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6})$

Table 1: Chiral Spectrum from Intersecting D6-branes.

ensures that the total angle of rotation is an element of $SU(3)$. The angles θ_i can be expressed in terms of the wrapping numbers as

$$\sin \theta_i = \frac{2^{-\beta_i} l^i R_2^i}{L^i(n^i, l^i)}, \quad \cos \theta_i = \frac{n^i R_1^i}{L^i(n^i, l^i)},$$

where R_1^i, R_2^i are the radii of the horizontal and vertical directions of the ith torus, and $L^i(n^i, l^i) = \sqrt{(2^{-\beta_i} l^i R_2^i)^2 + (n^i R_1^i)^2}$ is the total length of the one-cycle on the ith torus.

3.2 Search Strategy and Results

In [10], Cvetič, Papadimitriou, and Shiu (CPS) have performed an extensive search for $\mathcal{N} = 1$ supersymmetric three-family $SU(5)$ Grand Unified Models using the above construction. Therefore, we have used their work as the starting point for our search for the Pentagon model. In this section we will discuss what adjustments must be made to the CPS models in order to accommodate the inclusion of the Pentagon $SU(5)$ gauge group and our required matter multiplets. Based on simple assumptions that these adjustments lead us to, the existence of the Pentagon Model is ruled out. In particular, the number of stacks required to obtain the Pentagon spectrum introduces a problem with the complex moduli, and the simplest solution to this problem is not consistent with both tadpole and supersymmetry constraints. Relaxing these assumptions leads to models that are far more complicated and which must be evaluated on a case-by-case basis. Thus, while the construction of consistent models in the context of the Pentagon is not a forbidden possibility, it is left to future research.

We are looking to build a low energy phenomenological model that is 'at least as good' as the various CPS models, but with a few additional requirements. The CPS models are all four-dimensional chiral models with N=1 SUSY constructed from IIA

orientifolds on $T^2 \times T^2 \times T^2/(Z_2 \times Z_2)$. They satisfy consistency conditions (tadpole cancellation), preserve supersymmetry, and contain three generations of $SU_{GUT}(5)$ matter (or to be more precise, they all have 3 generations of the $\mathbf{10}_a$ representation of the $SU_{GUT}(5)$, but with varying number $\mathbf{5}$ fundamental representations). These models also have various phenomenological challenges, including the existence of substantial numbers of chiral exotics as well as issues with the Higgs fields and Yukawa couplings.

We are willing to accept these shortcomings for the present purpose, but there are other requirements that must be satisfied to reproduce the low energy spectrum of the Pentagon. Primarily, we require the existence of an additional stack of D-branes to give the $SU_P(5)$ of the Pentagon model, and total topological intersection number zero between this stack and the stack of the GUT $SU(5)$ (this is because of the vector-like nature of the Pentaquarks, which transform as either $(5,5)$ or $(5,\bar{5})$ plus c.c. under $SU_P(5) \times SU_{GUT}(5)$). This last requirement is satisfied by having the two stacks parallel on the first T^2 (by choice), but we wish to impose the additional constraint that the intersection number equal one on the remaining two Torii, so that there is only a single point at which the vector-like Pentaquarks may arise, thereby prohibiting additional unwanted generations of the Pentaquarks which could be disastrous for coupling unification and possibly introduce Landau poles. We also assume that the two parallel stacks on the first T^2 are actually lying right on top of each other, and further that they lie parallel to the orientifold plane. The first of these ensures that the pentaquarks remain massless, and the latter that they have no intersections with their orientifold images[4] (which would lead to exotic pentaquark-like fields charged under both $SU(5)$s). See figure 4.1. Finally, we would like to have two $U(1)$ stacks of D6-branes, the intersections of which would provide the singlets of the Pentagon. The desired particle content is summarized in table 2.

Let us begin by briefly listing the constraints relevant to our criteria. Following CPS, we define the parameters

$$A_a = -n_a^1 n_a^2 n_a^3, \ B_a = n_a^1 l_a^2 l_a^3, \ C_a = l_a^1 n_a^2 l_a^3, \ D_a = l_a^1 l_a^2 n_a^3$$

$$\tilde{A}_a = -l_a^1 l_a^2 l_a^3, \ \tilde{B}_a = l_a^1 n_a^2 n_a^3, \ \tilde{C}_a = n_a^1 l_a^2 n_a^3, \ \tilde{D}_a = n_a^1 n_a^2 l_a^3.$$

Then the tadpole cancellation conditions can be rewritten as

$$-16 = -2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a$$

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[4]Of course, the $SU(5)$ stacks and their images will still be parallel, so their positions must be fixed at positions on the first torus such that string states stretching between a brane and its image are massive, i.e. non-zero distance.
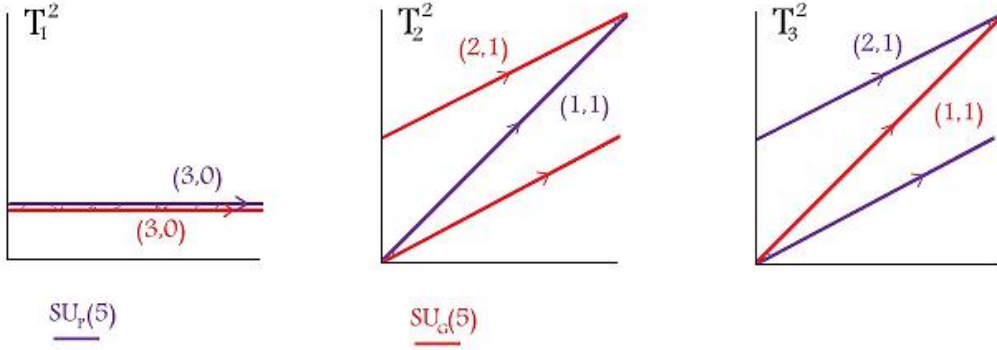
Figure 1: Geometeric Requirement for the Pentaquarks. Total intersection number is zero, with only a single intersection point (at the origin) in the second and third T^2.

Stack	Gauge Group	D6-brane Stack
a:	Pentagon $SU(5)$	$N_a = 10$
b:	SM GUT $SU(5)$	$N_b = 10$
c:	$U(1)$	$N_c = 2$
d:	$U(1)$	$N_d = 2$

$SU_P(5) \times SU_{GUT}(5)$ Particle Representation	Topological Intersection (Multiplicity)
$(5, \bar{5}) + (\bar{5}, 5)$	$I_{ab} = 0$
$(1, 10_a)$	$\frac{1}{2}(I_{bb'} + \frac{1}{2}I_{bO6}) = 3$
$(1, \bar{5})$	$I_{bc} + I_{bc'} + I_{bd} + I_{bd'} = -3$
Singlets	$I_{cd} + I_{cd'} \neq 0$

Table 2: Summary of Pentagon model D6-brane content and corresponding topological intersection numbers.

$$= -2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a.$$

The $N^{(i)}$ correspond to the number of 'filler branes' wrapping the ith orientifold plane, which is for our purposes arbitrary and can be used to reduce the total tadpole charge to the desired -16. They always contribute negatively, so our biggest obstacle will be to ensure that the sum of the charges from our stacks be large enough. The number of branes in each stack determines the gauge group, so in our case we want $N_a = 10, N_b =$

$10, N_c = 2, N_d = 2$. The extra factor of two is due to the orientifold projection. This allows us to write out the tadpole constraints more explicitly:

$$10A_a + 10A_b + 2A_c + 2A_d = -16 + 2^k N^{(1)}$$

or to simplify things

$$10A_a + 10A_b + 2A_c + 2A_d > -16$$

and similarly for B, C, D.

The supersymmetry constraints must be satisfied for each stack individually. The condition $\theta_1 + \theta_2 + \theta_3 = 0 \mod 2\pi$ is equivalent to $\sin(\theta_1 + \theta_2 + \theta_3) = 0$ and $\cos(\theta_1 + \theta_2 + \theta_3) > 0$, which can be rewritten in terms of our new variables as

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0$$

$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0$$

with similar expressions for stacks b,c,d. The x_A, x_B, x_C, x_D are related to the complex moduli of the tori $\chi_i = (R_2/R_1)_i$. Only three are independent (for simplicity we can set $x_A = 1$), and each must be positive. While we are free to adjust the moduli, each stack of branes introduces a new constraint. Thus generically three stacks of branes completely fix the three moduli of the tori, and so there is no freedom in adding a fourth stack of branes. For this reason, CPS only consider configurations with up to three non-trivial stacks, and this is a significant problem which must be addressed in our model.

CPS classify the possible brane wrapping configurations into four types, based on the number of tori in which the stack of branes is parallel to one of the orientifold planes (i.e. the number or ns or ls equal to zero). Type I has 3 zeros and so is completely parallel to one of the orientifold planes, these are the so-called 'filler branes'. Type II has two zeros, there are no SUSic configurations with two zeros. Type III has one zero, so a type III stack is parallel to the orientifold plane in one of the three tori. In this case, exactly two of A, B, C, D and two of $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}$ are zero. Without loss of generality we can choose $n^1 = 0$, so that $A = B = \tilde{C} = \tilde{D} = 0$, $CD = -\tilde{A}\tilde{B}$, and imposing the SUSY conditions we find

$$C < 0, \ D < 0, \ \tilde{A}\tilde{B} < 0, \ x_B = -\tilde{A}/\tilde{B}.$$

Finally, type IV has no zeros, and so $A\tilde{A} = B\tilde{B} = C\tilde{C} = D\tilde{D} = \text{constant} \neq 0$. Also, SUSY requires that only one of A, B, C, D is positive, and

$$x_A/A_a + x_B/B_a + x_C/C_a + x_D/D_a = 0.$$

Assuming a maximum of three non-trivial stacks of branes, CPS show that the $SU_{GUT}(5)$ brane stack must be type III, and taking $n_a^1 = 0$ they find $C_a = -1, D_a < -4$ with $k = 1$ or $k = 2$. Tadpole conditions then require a second stack with $D_b > 0$ and must therefore be type IV, with A_b, B_b, C_b negative. We still have the freedom to add a third stack of either type III or type IV, with the requirement $C_c \leq 0$.

In our case, since we are interested in four stacks, we would like two of the stacks to obey the same equation for the moduli, i.e. to have equal angles with respect to the orientifold plane. The natural choice then seems to be the GUT and Pentagon stacks since they must be parallel anyway. Our strategy then will be to consider the two $SU(5)$ stacks to be parallel in the first torus, but with the orientation of one stack in the second torus parallel to that of the other stack in the third and vice versa (see figure 4.1). So for example, two stacks with the winding numbers

$$SU_P(5) : (0, 3) \times (1, 1) \times (2, 1)$$

$$SU_{GUT}(5) : (0, 3) \times (2, 1) \times (1, 1)$$

would obey the same moduli equations; that is, the supersymmetry constraints on these stacks fix only one of the three moduli because the total angle of the stacks are the same. You will notice that this configuration has the two stacks parallel in the first torus (and therefore total topological intersection number of zero), while the number of intersection points in the last two tori is one as we would like, and if you calculate the number of chiral fields in the antisymmetric representation of the $SU_{GUT}(5)$ you will find the desired three families. In this example we would have $C = -3, D = -6, \tilde{A} = 3, \tilde{B} = -6, x_B = 1/2$ for the Pentagon stack (stack a), and $C = -6, D = -3, \tilde{A} = 3, \tilde{B} = -6, x_B = 1/2$ for the GUT stack (stack b). Unfortunately, this model is just one example of an entire class of similar models which suffer an incompatibility between the tadpole constraints and the SUSY conditions, as follows.

We have mentioned that stacks a and b must be parallel to the orientifold plane in order to avoid pentaquark-like exotics. This follows from the fact that we have demanded the number of intersection points in the second and third tori to be exactly one. If the two stacks were *not* parallel to the O6-plane in T_1^2, stack a would certainly intersect with the image of stack b in that torus (with the reverse being true as well). We might have hoped that the topological intersection could still be zero if there is a cancellation $(n_a^i l_b^i + n_b^i l_a^i) = 0, i = 2$ or 3, but this cannot be true if $(n_a^i l_b^i - n_b^i l_a^i) = 1, i = 2, 3$. Since n, l are integers, there is no way to add two numbers to get one and subtract them to get zero. Thus, both the Pentagon and GUT stacks must be type III.

However, if this is true, we cannot simultaneously satisfy the tadpole and supersymmetry conditions. Let us enumerate some of the requirements on a and b if they

are both to be type III. First, since the number of antisymmetric tensor representations for stack a is given by $I_{aa'} + 1/2I_{aO6} = 3$, and for type III $I_{aa'} = 0, I_{aO6} = 2^{3-k}(\tilde{A} + \tilde{B})$, we find that either $k = 1$ with $\tilde{A} + \tilde{B} = 3$ or $k = 2$ with $\tilde{A} + \tilde{B} = 6$, and in each case $\tilde{A}\tilde{B} < 0$ as before. Second, if we are to have the two stacks parallel in the first torus (by choice), we must have either $n_a^1 = n_b^1 = 0$ or $l_a^1 = l_b^1 = 0$ in order for them to satisfy the same moduli equations. We will choose the former for convenience. Finally, the requirement that we have only one intersection between stack a and b in the last two tori implies $n_a^2 l_b^2 - l_a^2 n_b^2 = 1, n_a^3 l_b^3 - l_a^3 n_b^3 = 1$. These conditions are solvable yet very confining, the simplest solution of which was given in the example above.

The problem then is this. We know that the values of C and D are both negative integers for both stacks a and b, and in fact $C_a = D_b, C_b = D_a$ for the type of configurations where stack a and b obey the same moduli equations as suggested above. This alone already implies that $C_a + C_b = D_a + D_b < -2$, but if the stacks are to satisfy the requirements of the previous paragraph the statement is more severe with $C_a + C_b = D_a + D_b < -9$. Recall that the tadpole condition instructs us to multiply these factors by the number of membranes in each stack, which again is $N_a = N_b = 10$, so that at best we have $-90 + 2C_c + 2C_d > -16$ and similarly for D. In other words, either C_c or C_d as well as D_c or D_d must be large and positive. This immediately rules out the possibility that stacks c and d are type III, because we know that for type III C and D are less than or equal to zero. For type IV stacks, only one of A, B, C, D can be positive and still satisfy SUSY conditions, so our only hope is that say $C_c > 0$ and $D_d > 0$ and that both values are large. Unfortunately, even this doesn't work. If $C_c > 0$ then D_c will contribute negatively to the D tadpole conditions, so of course we must require $|D_d| > |D_c|$, similarly $|C_c| > |C_d|$. This then leads to a problem with the moduli. For stacks c and d we have

$$A_c + x_B/B_c + x_C/C_c + x_D/D_c = 0$$

$$A_d + x_B/B_d + x_C/C_d + x_D/D_d = 0.$$

We have already solved for x_B previously (it is positive), so the the first two terms in each of these equations sum to a negative number. Multiply the equations by $C_c D_c$ and $C_d D_d$ respectively, and we can rewrite these as

$$|D_c|x_C - |C_c|x_D = P$$

$$-|D_d|x_C + |C_d|x_D = Q,$$

where P, Q are positive numbers. Summing the two equations we find

$$(|D_c| - |D_d|)x_C + (|C_d| - |C_c|)x_D = P + Q$$

implying that at least one of x_C or x_D is negative. This argument is analogous to the arguments in CPS given to forbid case (iv), $k = 2$ and case (i), $k = 3$ for type IV branes and case (i) for type III branes. Therefore, stacks a and b cannot be required to solve the same moduli equation. In fact, the argument is even stronger: stacks a and b cannot both be type III. This implies the existence of pentaquark-like exotics.

What if we relax this last requirement, i.e. not demanding stacks a and b to be type III? In order for two stacks to solve the same moduli equations, they must both be of the same type, so the question leads us to consider the compatibility of two stacks of type IV branes. The answer in this case is simple, and in fact applies regardless of the gauge groups supported on the stacks or their intersection number. As we have seen, the moduli equations for type IV can be written

$$A + x_B/B + x_C/C + x_D/D = 0,$$

and if any two stacks are to both obey the same equation we must have $A_1 = A_2, B_1 = B_2, C_1 = C_2, D_1 = D_2$. In this case, as far as the tadpole conditions and supersymmetry constraints are concerned, we can then just consider these two stacks as a single stack with $N = N_1 + N_2$ and $A = 2A_1$, etc. But we already know the requirements for a consistent model with three stacks. In particular, the GUT stack must be type III by the requirement of correct family multiplicity. Therefore, the possibility of the $SU_{GUT}(5)$ and $SU_P(5)$ stacks solving the same moduli equation is completely excluded. If we wish any other combination of type IV branes to solve the same moduli equation, the problem again reduces to the discussion of three stacks[5].

We are left with two possible approaches. The first would be to return to the possibility of constructing a consistent model with three stacks of D6-branes. We would then have to either assume that two of the stacks exactly obey the same moduli, supersymmetry, and tadpole equations (as suggested above), or to abandon one of the $U(1)$ stacks and argue that the Pentagon singlets arise from another mechanism. However, the question of $SU(5)$ GUT theories with three stacks of branes was exactly the subject of the CPS search. The GUT stack must be type III, and a second stack must be type IV. In their paper, they have listed all 149 possible solutions for models with a third stack of type III, none of which contain a second $SU(5)$ gauge group. Thus, the $SU_P(5)$ and any $U(1)$ factors must arise from stacks of type IV. According

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[5]This also provides an alternative argument proving the existence of pentaquark-like exotics in these models. If stacks a and b are both type III, they cannot be required to solve the same moduli equations. Because two stacks cannot solve the same moduli equation if they are of different types, this responsibility falls on the type IV stacks c and d. As argued, we can then consider these as a single stack. But we know that the Pentagon does not exist in a model with three stacks, two of which are type III. At least one of stacks a,b, then, must be type IV.

to CPS, this would require a very extensive search that would have to be conducted on a case by case basis, a search that CPS didn't endeavor to attempt. In any case, we believe it likely that a proof could be constructed to show that this possibility is inconsistent due to the severity of the constraints imposed by adding a second $SU(5)$ gauge group. This will be the subject of a future investigation.

The second possible approach is to allow a different combination of stacks to obey the same moduli equation. As we have argued, both these stacks would have to be type III or else the problem again reduces to a question of three stacks. We know that at minimum one of the stacks has to be type IV to satisfy the tadpole conditions, and this stack would likely have to sustain the $SU_P(5)$. If we make this assumption, we would have to find a non-trivial combination of two stacks with $N_a = 10$ and $N_b = 2$ which satisfy the same moduli equation. The argument would then parallel that of two type III stacks given above, but with some of the assumptions made there relaxed. If such a search were to fail, we would have to completely abandon the hope that two of our four stacks exactly solve the same moduli equation, and would be forced to find a system of equations in which the fourth stack obeys an equation which is a non-trivial linear combination of the other three. We have not yet found a strategy for systematically attacking this problem.

In any case, we know that these models will contain many undesired chiral exotics. CPS have shown that the existence of 15_{sym} representations are unavoidable in models with 10_as. We have further demonstrated that our models will contain chiral pentaquark-like exotics, charged as $(5, 5)$ under the Pentagon and GUT gauge groups. These particles are surely phenomenologically untenable.

Furthermore, though our search for the particle content of the Pentagon has proven somewhat inconclusive to this point, we find it likely that no self-consistent solution exists. The major culprit for this difficulty seems to be the tadpole constraints imposed by requiring two $SU(5)$ gauge groups. Generally speaking, the larger the gauge group, the more negatively a stack will contribute to the RR-charge. This fact, combined with the requirement that we find one (and only one) vector-like pair of pentaquarks, seems to be too great an obstacle to overcome. This leads us to believe that these constraints might be softened if we were to consider searching for the particle spectrum of the Pyramid model, for which we would need an $SU(3)^4$ gauge group arising from four stacks of $N = 6$ D6-branes. At the current time we are in the preliminary stages of such a search, and the approach seems promising.

4. M-theory on G_2 Manifolds

Because they carry no fluxes or additional charge sources, D6-branes and O6-planes are

seen to be pure geometrical artifacts in the strong coupling limit. This suggests that one might consider an M-theory description of chiral particles arising at points in the manifold. In particular, we are led to believe that $\mathcal{N} = 1$ globally supersymmetric type IIA intersecting D6-brane models lift up to eleven dimensional M-theory compactifications on singular G_2 manifolds [11]. D6-branes and O6-planes wrap smooth supersymmetric three cycles in the IIA compactifications, and one fibers each of these by a suitable noncompact hyperkähler four-manifold to obtain the G_2 holonomy space. In the M-theory language, these are codimension four ADE-orbifold singularities spanning three cycles in the G_2 compactification manifold, and must be ALE (asymptotically locally Euclidean) spaces. N overlapping D6-branes correspond to an A-type ALE singularity, D-type singularities arise for D6-branes overlapping O6-planes. Chiral fermions exist at isolated co-dimension seven singularities, which would correspond to the G_2 lift of the intersection points of D6-branes and O6-planes in the IIA picture. Just as in the IIA constructions, family replication is given by the number of these singular points in the manifold. When a point on the manifold shrinks to a conical singularity, the symmetry supported along that fiber will be enhanced at the singularity. To determine the chiral representations arising there, we decompose the adjoint of the group associated with higher symmetry with respect to that of the lower[12].

Specifically [13], we will obtain chiral fields in the representation R of group G if at certain points on the manifold the G singularity is enhanced to a group $\hat{G} = G \otimes U(1)$. Away from these points, the Lie algebra of $\hat{G}$ will decompose as

$$\hat{g} \rightarrow g \oplus o \oplus r \oplus \bar{r}$$

where g and o are the Lie algebras of G and $U(1)$, r transforms as R (and of charge 1 under $U(1)$), and $\bar{r}$ the complex conjugate. However, Acharya and Witten have shown that the net number of chiral zero modes is one, meaning that only either r or $\bar{r}$ will appear in the low energy theory (depending on how the chirality is fixed)[6]. The group G need not be simple, it may be any semi-simple product of the groups obtained by deleting one node from the Dynkin diagram of $\hat{G}$. The representations found at a particular singularity will not always be free from anomalies; however one can show that when this is the case there must exist another point (or set of points) elsewhere on the manifold supporting particles which render the theory anomaly free.

For A and D type singularities we are led to the representations listed in Table 3. Note that these are in agreement with the picture we have from IIA intersecting D6-brane models. For two three-cycles intersecting at a singular point in the G_2 manifold, supporting gauge groups $SU(N)$ and $SU(M)$ respectively, we are left with chiral

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[6]The discussion is complicated in the case of semi-simple G, see [14]

Resolution	Chiral Representation
$SU(N+1) \to SU(N)$	$[N]$
$SU(N+M) \to SU(N) \times SU(M)$	$[N, \bar{M}]$
$SO(2N) \to SU(N)$	$[N \otimes N]_a$
$SO(2(N+1)) \to SO(2N)$	$[2N]$

Table 3: Resolutions at A and D type singularities on manifolds of G_2 holonomy.

fermions in the bifundamental representation $(N, \bar{M})$. This is just as we would expect from the intersection of two stacks with N and M D6-branes. Similarly, the resolution $SO(2N) \to SU(N)$ leads to the antisymmetric representation of $SU(N)$, corresponding to particles which would be found at the intersection of a stack of N D6-branes with an O6-plane. However, the parallel should not be taken too literally. Unlike the IIA picture, three cycles in a seven-manifold do not generically intersect, so the existence of multiply charged particles will only be found in specially constructed geometries.

Can the Pentagon model be embedded in a G_2 compactification of M-theory? To answer this, we must find a 4-d theory with an $SU_P(5) \times SU_{SM}(5)$ gauge group and chiral fermions in the representations $(5, \bar{5})$ and $(\bar{5}, 5)$ (or possibly $(5, 5)$ and $(\bar{5}, \bar{5})$), $3 \times (1, 10)$, $3 \times (1, \bar{5})$, a pair of Higgs $(1, 5), (1, \bar{5})$, and a singlet field S (which we will ignore for the moment–we might assume it arises as some modulus of the geometry); all of which arise at singularities in the G_2 manifold. There is no single point that can sustain a symmetry which unfolds to the $SU(5) \times SU(5)$ gauge group plus all of the desired matter, so our model will necessarily have to be a patchwork of fields lying at different points in the manifold. This is not necessarily a problem; such a geometry would surely be less generic, but it could help explain the large family hierarchies of the standard model. The proximity of matter multiplets to Higgs fields would vary from point to point, creating a natural hierarchy in the Yukawa couplings.

It is clear that the desired components of our model can be derived in this construction, and our search for such a model in the IIA context points to the answer. Consider a three-cycle in the G_2 manifold sustaining an $SU(5)$ ADE-orbifold singularity. If at certain points along this cycle we find a conical singularity, the symmetry will be enhanced. If the enhanced gauge group is $SU(6)$, the symmetry will unfold as $SU(6) \to SU(5) \oplus o \oplus 5 \oplus \bar{5}$. Let us suppose the zero modes are the $\bar{5}$s, and that there are four of such points. Anomaly cancellation ensures that elsewhere on the manifold we can find representations of opposite chirality, but let us assume we find only one such point (leaving us with one 5 representation). Let the additional anomalies be canceled by 10_a representations, arising at singularities where the symmetry has been

enhanced to $SO(10)$, i.e. $SO(10) \to SU(5) \oplus o \oplus 10 \oplus \bar{10}$. Now assume that there exists an additional three-cycle on the manifold supporting a new $SU(5)$ symmetry, and that these two cycles somewhere intersect at a point. If this special point happens to lie at a conical singularity, the symmetry will be enhanced to $SU(10)$, which will resolve as $SU(10) \to SU(5) \times SU(5) \oplus o \oplus (5, \bar{5}) \oplus (\bar{5}, 5)$. Here we will find the pentaquarks, and elsewhere on the manifold we must find the anti-pentaquarks to cancel anomalies. Thus, this G_2 manifold will support two $SU(5)$ symmetries as well as the entire matter content of the Pentagon model, with no exotics (see Table 4).

Location	Supported Gauge Group	Enhanced Singularity	Matter Content
Three-Cycle 1	$SU(5)$	$5 \times SU(6)$	$4 \times \bar{5}$
			1×5
		$3 \times SO(10)$	$3 \times 10_a$
Three-Cycle 2	$SU(5)$		
Intersection	$SU(5) \times SU(5)$	$2 \times SU(10)$	$[5, \bar{5}]$
Points			$[\bar{5}, 5]$

Table 4: M-theory model containing the Pentagon spectrum.

Clearly, such a model will be highly non-generic. We may desire a model which sustains (at minimum) a pervasive symmetry $G = SU(5) \times SU(5)$ throughout the entire manifold, allowing the gauge group to be defined throughout the bulk. However, such a requirement complicates the model significantly. As we have seen, the pentaquarks can be found at points where the symmetry is enhanced to $SU(10)$, but this leaves no freedom to derive the chiral fields of the model[7]. On the other hand, the $SU(5) \times SU(5)$ gauge group can be obtained from Higgsing an $SU(10)$ with Wilson lines, so let us have the G_2 manifold support a pervasive $G = SU(10)$.

One possibility is that G uplifts to $\hat{G} = SU(11)$, in which case $\hat{g} \to su(10) \oplus o \oplus 10 \oplus \bar{10}$. With Wilson lines, the 10 will further decompose as $(1, 5) + (5, 1)$. Five of these points (with the proper chirality) will provide us with the standard model $3 \times (1, \bar{5})$ as well as the Higgs fields. However, we are left with some potentially undesirable particles, namely the four $(\bar{5}, 1)$s and the $(5, 1)$. We can imagine that the $(5, 1)$ will mass up with one of the $(\bar{5})$s, but we are still left with $3 \times (\bar{5}, 1)$. These particles are not necessarily problematic, as they have no standard model quantum numbers or

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[7]There may be an exception to this statement. We are currently investigating the possibility of singularities enhanced to $SU(6) \times SU(5)$ or $SO(10) \times SU(5)$. However, it is unclear to us at the present moment whether these fields will be charged under the second $SU(5)$ or have other undesirable $U(1)$ charges.

interactions. In fact, they may have the potential of providing us with a dark matter candidate. For this to be possible, we would need to find λ^{ijk}s (where $\lambda^{ijk}10^i\bar{5}^j\bar{5}^k$) such that the $U(3) \times U(3)$ flavor symmetry is broken to a conserved $U(1)$ with $Tr[T^3] = 0$. For now, let us just assume that these fields pose no phenomenological problems for the model.

If at another point we find $\hat{G} = SO(20)$, we would be left with $SU(10) + o + 45 + \bar{45}$ where the 45_a is the antisymmetric tensor representation of $SU(10)$. Higgsing the $SU(10)$, the 45 decomposes as $(5,5) + (1,10) + (10,1)$ under $SU_P(5) \times SU_G(5)$, leaving us with candidates for the pentaquarks and the standard model 10_a. However, in order to have three standard model generations there must be three separate points on the manifold with a $\hat{G} = SO(20)$ singularity. Thus we obtain (one half of) the desired pentaquarks (charged as $(5,5)$ as opposed to $(5,\bar{5})$), as well as the $3 \times (1,10)$ of the standard model; we also are left with an undesirable two additional copies of $(5,5)$ and with three $(10,1)$. The latter of these is possibly interesting (as discussed above), but the former spells disaster. Fortunately, with proper Z_4 R-charge assignments (two of the three $(5,5)$ with R-charge 0 and one with charge 2), we might imagine one pair massing up and leaving us with just the single generation of pentaquarks. Furthermore, this Z_4 could conceivably ensure that the 10s do not gain mass.

Of course, so far this model is not anomaly free. This is perhaps fortunate, because we are guaranteed to find the vector-like pair for the pentaquarks elsewhere on the manifold. We might be tempted to think the anomalies of the numerous 5 and 10 representations exactly cancel (there are equal numbers of $\bar{5}$s with $5 + 10_a$s), but there would then be no way to construct the anti-pentaquarks $(\bar{5},\bar{5})$. Thus we are forced to consider an additional three points elsewhere in the manifold supporting $\hat{G} = SO(20)$ singularities giving rise to the conjugate pairs. But this would lead to vector-like pairs $10 + \bar{10}$, which is clearly unacceptable. This would also force us to include additional 5 representations to cancel the anomalies of the $\bar{5}$s, and whether or not these pairs gain mass or remain light, this is certainly phenomenologically untenable.

One potential solution to this problem would be to soften our requirement for a pervasive $SU(10)$ gauge symmetry to a pervasive $SU(5) \times SU(5)$, but containing an entire fiber with enhanced symmetry $SU(10)$. The pentaquarks of this model would arise at points away from this fiber, but also enhanced to an $SU(10)$ symmetry, and unfolding as $SU(10) \to SU(5) \times SU(5) \oplus o \oplus (5,\bar{5}) \oplus (\bar{5},5)$ plus its complex conjugate. Along the fiber, certain points would have 'worsened' singularities with the enhanced symmetries $\hat{G} = SO(20)$, $SU(11)$. This would give rise to all of the desired components listed above, with the anomalies of the $\bar{5}$s and 10_as exactly canceling. The $(5,5)$s arising from the $SO(20)$ conical singularities would be exotics in this model. However, if for every $(5,5)$ representation there were an additional five singularities with $SU(11)$

symmetry, each producing a $(\bar{5}, 1) + (1, \bar{5})$ representation, the anomalies of these undesired particles would cancel. Furthermore, with correct R-charges, we might hope that all of these exotics gain mass. Of course, this model seems no more aesthetically viable than those without a pervasive symmetry listed above.

We therefore believe that the best candidate for our model is a G_2 manifold supporting two three-cycles with $SU(5)$ gauge symmetry intersecting at exactly two points. One of these cycles will support the standard model GUT $SU(5)$, and there will be points along this cycle at which the symmetry is enhanced to either $SU(6)$ or $SO(10)$ giving rise to the standard GUT matter. The intersection points must lie at special points on the manifold where the K3 structure has an enhanced symmetry, such that we find $SU(10) \rightarrow SU(5) \times SU(5) \oplus o \oplus (5, \bar{5}) \oplus (\bar{5}, 5)$ at one intersection and the vector-like partners at the other. Though this type of geometry is certainly highly non-generic, it arises naturally from a lift of type IIA intersecting D6-branes. In such a model, we expect to find chiral particles in the bifundamental representation exactly at such points where two three-cycles intersect. Indeed, the fact that such a construction is possible in the M-theory context suggests that we might hope to find a consistent model of intersecting D6-branes. Conversely, the difficulty we have found in constructing such models might suggest that the existence of such an M-theory geometry is dubious. Unfortunately, the tools necessary to perform an explicit calculation are unknown, and the existence of such a model remains in question.

5. Heterotic Orbifold Constructions

Some of the most realistic phenomenological string models have been produced in the framework of heterotic orbifolds[15]. This presents us with a promising approach toward discovering a phenomenologically viable low energy model. In this section we will briefly review the heterotic orbifold construction[8], and then proceed to discuss our search strategy and preliminary results.

5.1 Brief Review

Heterotic string theory is a theory of closed strings, combining the supersymmetric right moving string with the left moving bosonic string. As the (uncompactified) theory must exist in 10 space-time dimensions, the extra 16 dimensions of the bosonic string are interpreted as internal degrees of freedom. To satisfy modular invariance (and anomaly cancellation) these 16 left movers must live on a 16 dimensional Euclidean even self-dual lattice, which we choose to be the root lattice of $E_8 \times E_8$ (although the $SO(32)$ root lattice has been shown to be of interest as well [17]).

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[8] In addition to the references given in [15], see [16] for further details

The low energy effective field theory will consist of those states which survive to low energies. In particular, any state with mass at the string scale will not be observed at low energy, so we are only concerned with string states of zero mass. Further, since the physical heterotic string states are the direct product of the right movers with the left movers, both the right-moving and left-moving string states must be massless. Working in the light cone gauge, we find that there is a total of 16 massless right movers, 8 in the NS sector which transform as an $SO(8)$ vector and 8 in the R sector transforming as the $SO(8)$ spinor. When tensored with the left movers, the vector representation will produce the boson of the 10 dimensional supersymmetric chiral fields, while the spinor gives its fermionic superpartner. The 8 bosonic left moving oscillators corresponding to the space-time degrees of freedom create massless states when acting on the left moving ground state; when tensored with the right movers, these form the $\mathcal{N} = 1$ supergravity multiplet. Similarly, the 16 internal degrees of freedom bosonic oscillators acting on the left moving ground state form the 16 uncharged gauge bosons of $E_8 \times E_8$ (and their superpartners) when tensored with the right movers. Finally, there are the massless 240 + 240 charged gauge bosons (plus superpartners) of $E_8 \times E_8$, which come from the tensor product of the right movers with those left moving states having internal momenta satisfying $(p^I)^2 = 2$. This is exactly the condition for the root vectors of E_8, ensuring that these states lie on the $E_8 \times E_8$ root lattice. Altogether, the massless heterotic string states form a ten-dimensional $\mathcal{N} = 1$ supergravity theory with $E_8 \times E_8$ gauge group.

As the Pentagon model is a four dimensional low energy effective field theory, six of the space-time dimensions must be compactified. The simplest way to achieve this is to wrap each of these extra coordinates on a circle, which is topologically equivalent to compactifying on the 6-torus T^6. However, torus compactification schemes in general do not lead to realistic models in four dimensions. In particular, the $SO(8)$ spinor of the 10 dimensional heterotic theory compactified on T^6 gives a total of 4 gravitinos in 4 dimensions, thus leading to $\mathcal{N} = 4$ supersymmetry. To obtain a chiral theory with $\mathcal{N} = 1$ supersymmetry, one may compactify on an orbifold

$$\mathcal{O} = T^6/\mathcal{P} \otimes T_{E_8 \times E_8}/\mathcal{G},$$

where the space-time and internal degrees of freedom are differentiated to admit a clear space-time interpretation. Formally, an orbifold is defined to be the quotient of a torus over a discrete set of isometries of the torus, called the point group $\mathcal{P}$. The simplest of these is the symmetric abelian orbifold, where the point group is chosen to be the cyclic group Z_N with $N = 3, 4, 6, 7, 8, 12$. The lattice on which $\mathcal{P}$ acts as an isometry will be the root lattices of semi-simple Lie algebras of rank 6. The space group $\mathcal{S}$ is defined to be the point group $\mathcal{P}$ plus the translations given by these lattice vectors,

such that $T^6/\mathcal{P} = R^6/\mathcal{S}$. The action of the space group on the (complex) space-time degrees of freedom can be written as

$$Z^a \rightarrow e^{(2\pi i v^a)} Z^a + n_\alpha e^\alpha, \ a = 1, 2, 3$$

where v, the generator of the discrete group Z_N, is called the twist vector, and the e^α are the lattice vectors of the root lattice spanning T^6. Thus, two points on R^6 are identified if they differ by the action of the space group. Points that are invariant under the action of the space group are known as fixed points of the orbifold. To ensure that exactly one 4 dimensional space-time supersymmetry survives, $\pm v^1 \pm v^2 \pm v^3 = 0$ mod 2 with none of the v^a vanishing.

$\mathcal{G}$ is called the gauge twisting group. Modular invariance requires the action of the space group to be embedded into the gauge degrees of freedom. This means that in general the internal gauge group of the orbifold will be a subgroup of the $E_8 \times E_8$ gauge group of the uncompactified heterotic theory. To realize this embedding, the orbifold twist vector is associated with a shift vector V in the $E_8 \times E_8$ root lattice, while the torus shifts e^α are embedded as shifts W_α. Since the W_α correspond to gauge transformations associated with non-contractible loops, they are interpreted as Wilson lines. The action of the gauge twisting group $\mathcal{G}$ on the gauge degrees of freedom is

$$X^I \rightarrow X^I + 2\pi(kV^I + n_\alpha W_\alpha^I).$$

The combined action of $\mathcal{S} \otimes \mathcal{G}$ is known as the orbifold group.

Not all gauge twists and discrete Wilson lines are physically allowed. Modular invariance automatically guarantees the anomaly freedom of orbifold models. For the partition function to be modular invariant, it must satisfy the following conditions:

$$(V^2 - v^2) = 0 \text{ mod } 2$$

$$V \cdot W_\alpha = 0 \text{ mod } 1$$

$$W_\alpha \cdot W_\beta = 0 \text{ mod } 1, \ \alpha \neq \beta$$

$$W_\alpha^2 = 0 \text{ mod } 2.$$

These conditions are known as 'strong modular invariance'. In reality one need only satisfy 'weak modular invariance', where these conditions are slightly relaxed:

$$N(V^2 - v^2) = 0 \text{ mod } 2$$

$$NV \cdot W_\alpha = 0 \text{ mod } 1$$

$$NW_\alpha \cdot W_\beta = 0 \text{ mod } 1, \ \alpha \neq \beta$$

$$NW_\alpha^2 = 0 \bmod 2.$$

However, if weak modular invariance is satisfied, we can in general bring V and W_α to a form which obeys strong modular invariance by adding $E_8 \times E_8$ lattice vectors. This has the advantage of simplifying the projection conditions on physical states.[9] N is the order of the orbifold (and of the cyclic group Z_N). Cyclic group multiplication rules require that N successive rotations of the orbifold act as the identity $Nv = 0 \bmod 1$, and that NV belongs to the $E_8 \times E_8$ lattice.

The gauge transformations are required to be a symmetry of the system. To calculate which states survive orbifolding, we must consider the action these transformations have on the states with right- and left-moving momentum. Neither the shifts nor the twists act on the oscillators. The generator of translation is

$$e^{ip \cdot X}|0\rangle = |P\rangle,$$

so a shift in the coordinate degrees of freedom acts as a phase rotation on the states. For the right movers,

$$|q\rangle \to e^{2\pi i q \cdot (kv)}|q\rangle$$

and for the left movers,

$$|P\rangle \to e^{2\pi i p \cdot (kV + n_\alpha W_\alpha)}|P\rangle.$$

States that are invariant with respect to the orbifold group transform trivially (with a phase of 1) under every element of the group, i.e. for all $k, n_\alpha = 0, ..., N-1$. Only invariant states are consistent with the geometry of the underlying orbifold space; all other states must be projected out.

The massless spectrum consists of all massless closed string states consistent with the geometry of the orbifold. This includes the massless strings of the original heterotic theory which survive the projection conditions, as well as additional new states which arise due to the non-trivial geometry of the orbifold. The former form the untwisted sector and are free to move throughout the orbifold, while the latter are known as twisted sector states and are confined to the fixed points.

First consider the untwisted sector. As mentioned earlier, orbifolding projects out three of the supersymmetries, and we are left with an $\mathcal{N} = 1$ supergravity multiplet (as well as certain modulus fields which are not relevant to the current discussion). The 16 uncharged gauge bosons correspond to the Cartan generators of the $E_8 \times E_8$ algebra. By construction, the gauge twists and Wilson lines must commute with the Cartan subalgebra, thus all uncharged gauge bosons (and gaugino partners) survive the orbifold projection. Furthermore, the rank of the algebra can never be reduced by

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[9]There are exceptions to this rule, such as when $V = 0$.

the shift embedding. The charged gauge bosons of the heterotic string give rise to both the unbroken gauge group as well as charged matter states. As these are states with both right- and left-moving momenta, they transform under the orbifold group as

$$|q\rangle \otimes |P\rangle \to e^{2\pi i(q\cdot(kv)+p\cdot(kV+n_\alpha W_\alpha))}|q\rangle \otimes |P\rangle.$$

The momenta of the right movers are given by their $SO(8)$ weights:

$$q = (\underline{\pm 1,0,0,0}) \quad \text{bosons}$$

$$q = (\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2) \quad \text{fermions}.$$

The underline denotes that all permutations are included. Only states with an even number of minus signs are included for the fermions. Gauge bosons in four dimensions have two transverse polarizations, and so require oscillators in the uncompactified directions, i.e. $q = (\pm 1, 0, 0, 0)$ in common notation.[10] Similarly, the gaugino states must have the right movers $q = \pm(1/2, 1/2, 1/2, 1/2)$. Thus, the right movers of the four dimensional gauge bosons (and gauginos) are invariant under the orbifold action, $q \cdot v = 0$. The left movers, then, must satisfy

$$p \cdot (kV) = 0 \bmod 1$$

$$p \cdot (n_\alpha W_\alpha) = 0 \bmod 1$$

for all k, n_α. Not all of the charged gauge bosons of the heterotic string will satisfy these conditions, so the gauge group is broken; those that do survive (along with the 16 Cartan generators) form the generators of the unbroken gauge group on the orbifold. However, there are additional states which satisfy

$$q \cdot (kv) + p \cdot (kV + n_\alpha W_\alpha) = 0 \bmod 1$$

without fulfilling $q \cdot v = 0$. These states are interpreted as charged matter, and the root vectors p are their weights with respect to the unbroken gauge group.

A twisted string is one that closes only by imposing the space group symmetry,

$$Z^a(\tau, \sigma + 2\pi) = e^{(2\pi i k v^a)} Z^a(\tau, \sigma) + n_\alpha e^\alpha,$$

i.e. by performing both twists and lattice shifts. Thus, they must be localized at the fixed points. Each of these states is dependent on the required number of twists, thus k

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[10]In complex coordinates, the first component refers to the uncompactified directions and the last three components to the coordinates on the six-torus T^6. The lightcone coordinates are gauge fixed and are omitted. Typically, the first component is omitted when writing the twist vector, v, as it must be zero.

labels the $N-1$ twisted sectors ($k = 0$ corresponds to the untwisted sector). Similarly, the presence of Wilson lines is determined by the corresponding lattice shifts required at each fixed point. Wilson lines affect the mass equation for the left movers (as we will see below), so this has the effect of changing the representations found at different fixed points. Modifying the boundary conditions for the twisted sector changes the mode expansions for the right and left movers, which in turn shifts the weights of the states, $q \to q + kv$ and $p \to p + (kV + n_\alpha W_\alpha)$. As a result, the level matching condition for the massless states now reads

$$\frac{1}{2}(q+kv)^2 - \frac{1}{2} + \delta c = \frac{1}{4}m_R^2 = \frac{1}{4}m_L^2 = \frac{1}{2}(p+kV+n_\alpha W_\alpha)^2 + N_L - 1 + \delta c = 0.$$

N_L is the number operator for the left movers, and is allowed to be fractional as a consequence of a non-trivial twist. To be more specific, $N_L = \sum_a (\eta^a N_{La} + \bar{\eta}^a N_{La}^*)$, where $\eta^a = kv^a$ mod 1 with $0 \le \eta^a < 1$, $\bar{\eta}^a = -kv^a$ mod 1 with $0 \le \bar{\eta}^a < 1$, and N_{La}, N_{La}^* are oscillator numbers of the left movers in the z_a and $\bar{z}_a$ directions. δc is a shift in the zero point energy, and is given by

$$\delta c = \frac{1}{2}\sum_{a=0}^{3}\eta^a(1-\eta^a).$$

Once the massless spectrum of the twisted sectors is calculated, projection conditions must be applied. Among the massless representations, physical states are selected by the generalized GSO projection operator. In a theory with non-trivial Wilson lines, the momentum shift is dependent on the fixed point under consideration as discussed earlier. Therefore, the GSO projection should be applied to each state individually. This can be written:

$$P(k, \gamma, n_\alpha) = \frac{1}{N}\sum_{l=0}^{N-1}[\Delta(k, \gamma, n_\alpha)]^l$$

with

$$\Delta(k, \gamma, n_\alpha) = \phi\gamma e^{2\pi i[(P+kV+n_\alpha W_\alpha)\cdot(V+n_\alpha W_\alpha)-(r+kv)\cdot v]}$$

Here, k labels the twisted sector, and the n_α label the order of the Wilson lines relevant for the given state (corresponding to the number of lattice shifts required for the point to be invariant under the space group action). γ is the eigenvalue of the state under the action of k orbifold twists. For prime orbifolds (e.g. Z_3, Z_7) this factor is trivial, $\gamma = 1$. For non-prime orbifolds, physical states are defined by linear combinations of massless states living at fixed points which transform into each other under the space group action. These physical states can be shown to have definite eigenvalue $\gamma = e^{(2\pi q_\gamma)}, q_\gamma = 0, 1/n, 2/n...1$ under the rotation. The oscillator phase is

$$\phi = e^{2\pi i \sum_a v_a(N_{La}-N_{La}^*)}.$$

For any non-trivial phase Δ, the contributions of Δ^l in the sum for P will all add up to zero. Thus, only states satisfying $\Delta(k, \gamma, n_\alpha) = 1$ will survive the projection. Equivalently, the projection condition can be written:

$$(P + kV + n_\alpha W_\alpha) \cdot (V + n_\alpha W_\alpha) - (r + kv) \cdot v + \sum_a v_a(N_{La} - N_{La}^*) + q_\gamma = 0 \bmod 1.$$

For states with $q_\gamma = 0$ (i.e. for prime orbifolds), one can use the modular invariance equations to show that this condition is in fact automatically satisfied for all states satisfying the mass equation. Thus, all massless representations of the prime orbifolds are in fact physical states, and the GSO projector need not be calculated.

The above construction provides the rules for calculating the entire low-energy spectrum of the heterotic orbifold theory. For calculational convenience, we have automated the process using Mathematica, and included it as an Appendix[11]. The required input is simply the orbifold twist vector, the gauge shift, and the Wilson lines; the program will then check modular invariance, calculate the gauge group and output the surviving simple roots and Cartan matrix, and calculate the surviving states in both the untwisted and twisted sectors, displaying the highest weight representations in Dynkin label notation. The surviving gauge groups and representations may be interpreted by comparison with, for example, the extensive tables of [18].

5.2 Search Strategy and Results

Heterotic models based on Z_N orbifolds are well known and have been discussed extensively in the literature. There are a finite number of gauge groups obtainable from $E_8 \times E_8$ for a particular orbifold, and these have all been systematically classified and their matter contents calculated. In [19], the authors have tabulated the results for every inequivalent modular invariant gauge shift (with no Wilson lines) for each discrete orbifold. Wilson lines complicate the theory significantly, as they provide a mechanism to further break down the gauge symmetries of the models as well as to change the representations found at different fixed points, thereby greatly increasing the number of inequivalent models. Still, the rules are well understood and a large number of these models have been calculated. The prime orbifold Z_3 is particularly well known as it has the simplest transformation properties under the orbifold group. Therefore, we have chosen the Z_3 orbifold as the starting point for our search. Calculations of twisted sector states in Z_3 models are greatly simplified due to the fact that GSO projectors need not be calculated if strong modular invariance is satisfied. Conversely, the simplicity of the projectors in this case allow a straightforward calculation of physical states, sug-

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[11]The program does not implement GSO projectors, and these must be checked by hand.

gesting we may employ weak modular invariance to ease the constraints on our models. We have elected to follow the latter approach.

There are only five possible breakings of E_8 by $N = 3$ modular invariant gauge shifts (without Wilson lines): $E_6 \times SU(3)$, $SU(9)$, $E_7 \times U(1)$, $SO(14) \times U(1)$, and E_8 (unbroken). Clearly, the $SU(5)$ factors of the Pentagon model would have to arise from different E_8s, and there is only a limited number of Wilson lines that would provide the desired symmetry. However, there is a very large number of ways to fit $SU(3)^4$ into $E_8 \times E_8$. Furthermore, it would seem natural for the Z_3 symmetry of the trinification model to arise as the result of the geometry of the orbifold. Thus, we have elected to confine our search to the particle spectrum of the Pyramid model. That is, we wish to find the low energy gauge group $SU(3)^4$ with matter content

$$
\begin{aligned}
&3 \times (3, 1, \bar{3}; 1) + (1, \bar{3}, 3; 1) + (\bar{3}, 3, 1; 1) \quad &&\text{Standard Model Fermions} \\
&(3, 1, 1; \bar{3}) + (\bar{3}, 1, 1; 3) \\
&(1, 3, 1; \bar{3}) + (1, \bar{3}, 1; 3) \quad &&\text{Trianons} \\
&(1, 1, 3; \bar{3}) + (1, 1, \bar{3}; 3)
\end{aligned}
$$

on a Z_3 orbifold with twist vector $(1/3, 1/3, -2/3)$. The matter content of standard trinification fits naturally into a 27 representation of E_6. Thus, we will further assume that three of the $SU(3)$ factors fit into an E_6 subgroup of a single E_8. There is only one gauge shift that will break E_8 to $E_6 \times SU(3)$ on a Z_3 orbifold, $V = (2/3, 1/3, 1/3, 0, 0, 0, 0, 0)$ (there are other modular invariant gauge shifts that have the same effect, but they are all equivalent to the one listed by shifts in the lattice). Of course, in our models V has 16 components, 8 degrees of freedom corresponding to each E_8. The full vector must satisfy (strong) modular invariance, and the condition $(V^2 - v^2) = 0 \mod 2$ provides little freedom for the last eight components. Thus we will choose $V = (2/3, 1/3, 1/3, 0, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0)$, leaving the second E_8 unbroken for the moment. This will change with the addition of Wilson lines.

In fact, realistic trinification models have been discovered under these assumptions [20]. These models do not, unfortunately, exactly reproduce the spectrum of the Pyramid model. In particular, while some of these do include a fourth $SU(3)$ gauge group, none contain a full set of vector-like trianons. However, their models do provide useful guidelines for directing our search. There are only two models listed in the tables of [19] which give an $SU(3)^3$, but these do not have the correct matter content. Thus, one is forced to consider a model with Wilson lines. In [21] the authors have classified all possible Wilson line breakings of $E_6 \times SU(3)$ on a Z_3 orbifold with one Wilson line, and have tabulated the resulting gauge groups. There are only two possibilities (up to

lattice shifts) for obtaining $SU(3)^3$. They are

$$W_1 = (0, 2/3, 1/3, 1/3, 1/3, 1/3, 0, 0) \quad \rightarrow SU(3)^3 \times U(1)^2$$
$$W_2 = (5/3, 1/3, 1/3, 1/3, 1/3, 1/3, 0, 0) \rightarrow SU(3)^4$$

It would be convenient if the entire Pyramid model fit into a single E_8, with the second E_8 remaining hidden. For that to be true, the gauge shift would break E_8 to the Pyramid $SU(3)$ times the Standard Model GUT E_6, and Wilson lines (specifically W_2 above) would further break E_6 to the desired trinification $SU(3)$. Unfortunately, these choices do not produce the desired spectrum, and it appears that the Pyramid $SU(3)$ will have to arise from the second E_8. Phenomenologically this poses no problem; it does however make obtaining this model quite difficult, due to the fact that there are no fields charged under both E_8s in the non-orbifolded heterotic string theory. Since all chiral representations obtained in the untwisted sector are merely a subgroup of the entire $E_8 \times E_8$ adjoint which survive the projection conditions, it is impossible to obtain representations charged under both E_8s from the untwisted sector. However, because the momenta of the states existing at the fixed points of the lattice are shifted by the presence of Wilson lines, it is possible to obtain states charged under both E_8s in the twisted sector.

Thus, our search strategy has been as follows. We begin with the Z_3 orbifold obtained from the gauge twist $(1/3, 1/3, -2/3)$. We wish to break the first E_8 to $E_6 \times SU(3)$ via the gauge twist $V = (2/3, 1/3, 1/3, 0, 0, 0, 0, 0)(\overrightarrow{0})$. To obtain standard trinification with no chiral exotics, we must further break $E_6 \rightarrow SU(3)^3$ and $SU(3) \rightarrow U(1)^2$. This can be acheived by W_1 alone, or by a combination of W_2 and additional Wilson lines, such as $W_3 = (1/3, 1/3, 0, 0, 0, 0, 0, 0)$. Whichever we choose, we must also assign values for the eight additional components corresponding to the second E_8 such that the entire 16 component vector remains modular invariant. The second E_8 gauge group must be broken to $SU(3) \times G$, where G is some unspecified cofactor. Phenomenologically the group G is arbitrary as long as there are no fields charged simultaneously under both it and the trinification group.

Because the lattice vector e_1^i is equivalent to e_2^i on each two-torus T_i^2 by the action of the twist, the Z_3 orbifold can sustain a maximum of three Wilson lines, one for each two-torus. As we have discussed, the presence of a Wilson line differentiates states at different fixed points of the corresponding torus. Because there are 27 fixed points on the Z_3 orbifold, the multiplicity of twisted sector states in a model without Wilson lines would be 27. Models with one Wilson line will have twisted sector multiplicity 9, two Wilson lines give multiplicity 3, and three Wilson lines differentiates each fixed point individually. Models with two Wilson lines seem to suggest a geometric explanation for the family multiplicity of the Standard Model. However, we are constrained to find

only a single generation of the trianons, and are therefore led to consider models with three Wilson lines. This of course complicates the task of finding three trinification generations.

Thus far we have fixed the gauge twist and have narrowed the possibilities for one of the Wilson lines. There still remains the freedom to choose two additional Wilson Lines–each of which is a 16 dimensional vector. Constraints are imposed due to the fact that the Wilson lines must obey modular invariance and by the requirement that we do not break the gauge symmetry of the first E_8 beyond $SU(3)^3$. Nevertheless, this still permits a vast number of models to be calculated if we are to scroll through each possible vector in succession (perhaps on the order of $> 10^{10}$), making a comprehensive search rather difficult. At the present time we do not have the computing power necessary to perform such a search, though we would like to do so in the future. Actually, it is conceivable that the number of distinct Wilson lines is in fact much smaller due to the fact that many will be equivalent up to lattice shifts, but we have not found a way to use this fact to our advantage at the current time. Thus, to this point we have only endeavored to follow the more modest approach of trial and error.

Unfortunately, we have not found anything resembling the complete spectrum of the Pyramid model. While a large number of models contain the standard trinification spectrum (as we should expect considering we have specifically chosen our gauge twist and Wilson lines to enforce this), it is very difficult to obtain the trianons. We believe this is due to the difficulty of finding particles charged under both E_8s. It is interesting to note that the models we have found closest resembling the spectrum of the Pyramid contain a chiral set of the trianon-like particles, but finding their vector-like partners has proved elusive. This is not entirely surprising, considering that the heterotic orbifold models were originally constructed to produce a chiral spectrum. This could even be a general symptom of these models (and a failure for our purposes), but a comprehensive search would have to be conducted to know this for certain.

We present two interesting results in the tables 5, 6. The first model is perhaps the most promising. It contains three complete trinification generations, as well as a number of Higgs-like fields. It also contains a single generation of (an incomplete set of) chiral trianon-like particles, but it does not contain their vector-like partners. The model also contains a few chiral exotics. The second model is interesting in that it contains a single complete set of *chiral*-like trianons (i.e. one half of the 6 total). It also complains a completely *vector*-like trinification spectrum. It should be noted that the GSO projectors have not been implemented on the spectra of these models (the projectors will only project out states, and the spectra are incomplete to begin with), and the spectra listed are therefore anomalous.

The next step in our research will be to clearly establish the number of inequivalent

Gauge Group	$SU(3)^3$	$SU(3) \times SO(8)$
V	(2/3,1/3,1/3,0,0,0,0,0)	(0,0,0,0,0,0,0,0)
W_1	(0,2/3,1/3,1/3,1/3,1/3,0,0)	(2/3,0,0,0,0,0,0,0)
W_2	(0,2/3,1/3,1/3,1/3,1/3,0,0)	(1/3,1/3,1/3,1/3,0,0,0,0)
W_3	(1/3,0,1/3,0,0,0,0,0)	(1/3,1/3,1/3,1/3,0,0,0,0)

Spectrum	Particle	Vector Partner
Trianons	$(\bar{3}, 1, 1)(\bar{3}, 1)$	
	$(1, 3, 1)(3, 1)$	
Trinification	$3 \times (1, 3, \bar{3})(1, 1)$	
	$3 \times (3, \bar{3}, 1)(1, 1)$	
	$3 \times (\bar{3}, 1, 3)(1, 1)$	
Higgs	$13 \times (3, 1, 1)(1, 1)$	$7 \times (\bar{3}, 1, 1)(1, 1)$
	$7 \times (1, 3, 1)(1, 1)$	$13 \times (1, \bar{3}, 1)(1, 1)$
	$13 \times (1, 1, 3)(1, 1)$	$13 \times (1, 1, \bar{3})(1, 1)$
Exotics	$3 \times (1, 3, 3)(1, 1)$	
	$3 \times (\bar{3}, 1, \bar{3})(1, 1)$	
	$3 \times (3, \bar{3}, 1)(1, 1)$	
	$4 \times (1, 1, 1)(3, 1)$	
	$4 \times (1, 1, 1)(\bar{3}, 1)$	
	$4 \times (1, 1, 1)(1, 8)$	
Singlets	$17 \times (1, 1, 1)(1, 1)$	

Table 5: Z_3 heterotic orbifold model 1. Contains Standard Model trinification and chiral trianon-like particles

modular invariant Wilson lines for the Z_3 orbifold, and to perform a comprehensive search for the Pyramid model spectrum. If such a search fails to produce the desired spectrum, we will be forced to perform a similar search in the other Z_N orbifolds, probably forcing us to abandon our desire for the Z_3 trinification symmetry to be an artifact of the geometry of the manifold[12]. Regardless of the outcome, we are still interested in a future search for the $SU(5) \times SU(5)$ gauge group and particle spectrum of the Pentagon model.

6. Concluding remarks

Though we have not been able to rule out the existence of the Pentagon model as a

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[12]It might still arise as a result of a non-prime orbifold, $Z_6 = Z_2 \times Z_3$ or $Z_{12} = Z_3 \times Z_4$.

Gauge Group	$SU(3)^3$	$SU(3) \times E_6$
V	(2/3,1/3,1/3,0,0,0,0,0)	(0,0,0,0,0,0,0,0)
W_1	(5/3,1/3,1/3,1/3,1/3,1/3,0,0)	(2/3,1/3,1/3,0,0,0,0,0)
W_2	(2/3,1/3,1/3,0,0,0,0,0)	(2/3,1/3,1/3,0,0,0,0,0)
W_3	(1/3,1/3,0,0,0,0,0,0)	(2/3,1/3,1/3,0,0,0,0,0)
Spectrum	**Particle**	**Vector Partner**
Trianons	$(3,1,1)(3,1)$	
	$(1,\bar{3},1)(\bar{3},1)$	
	$(1,1,\bar{3})(\bar{3},1)$	
Trinification	$(1,3,\bar{3})(1,1)$	$(1,\bar{3},3)(1,1)$
	$(3,\bar{3},1)(1,1)$	$(\bar{3},3,1)(1,1)$
	$(\bar{3},1,3)(1,1)$	$(3,1,\bar{3})(1,1)$
Higgs	$7 \times (3,1,1)(1,1)$	$7 \times (\bar{3},1,1)(1,1)$
	$7 \times (1,3,1)(1,1)$	$7 \times (1,\bar{3},1)(1,1)$
	$7 \times (1,1,3)(1,1)$	$7 \times (1,1,\bar{3})(1,1)$
Exotics	$(1,3,3)(1,1)$	$(1,\bar{3},\bar{3})(1,1)$
	$(3,3,1)(1,1)$	$(\bar{3},\bar{3},1)(1,1)$
	$(3,1,3)(1,1)$	$(\bar{3},1,\bar{3})(1,1)$
	$4 \times (1,1,1)(\bar{3},1)$	
	$(1,1,1)(1,27)$	
	$(\bar{3},1,1)(3,1)$	
Singlets	$7 \times (1,1,1)(1,1)$	

Table 6: Z_3 heterotic orbifold model 2. Contains a *chiral* set of trianons and *vector*-like trinification.

low energy effective field theory embedded in a string theory, we have thus far had no success in constructing such a model. In each of the embedding structures we have explored, the constraints imposed by our criterion have proven to be quite strict. In part this is due to the size of the desired gauge groups, but we believe that an even more restricting constraint is the requirement that we find both chiral and vector-like particles in the spectrum.

This requirement posed a strict constraint on the geometry of the two stacks of D6-branes supporting $SU(5)$ gauge groups in the type IIA construction. In fact, we found that no such structure was able to satisfy the same equation for the complex structure moduli while remaining consistent with RR-tadpole charge cancelation. This problem translates into a difficulty with maintaining supersymmetry. While there may

exist a more complicated geometry satisfying all of our criteria, finding such a model proved to be beyond the scope of our current search. However, another approach we are currently investigating is to embed the Pyramid model into an intersecting D6-brane construction, though the results of this search are still unclear.

We did discover a potential candidate for the existence of the Pentagon model in the case of M-theory manifolds of G_2 holonomy, though the proof of its existence is beyond our capabilities. However, such a model does not support a pervasive $SU(5) \times SU(5)$ symmetry throughout the G_2 manifold. If we include this criteria as a requirement for the model, we have shown that it becomes quite difficult to obtain both the vector-like pentaquarks and the chiral antisymmetric 10 representations of the GUT $SU(5)$. This follows from the fact that both representations are found at a singularity which resolves as $SO(20) \rightarrow SU(10) + o + 45 + \overline{45}$. We may break the SU(10) via Wilson lines, leaving us with chiral particles in the representations $(5, 5) + (1, 10) + (10, 1)$. Therefore, in this construction, it is impossible to find vector-like partners for the pentaquarks without simultaneously producing vector-like partners for the 10s.

We have also shown that it is difficult to obtain the vector-like trianons of the Pyramid model in a heterotic orbifold construction. While we were able to find models with a standard trinification spectrum, we found that the trianons must arise in the twisted sector of a Z_3 orbifold due to the fact that they must come from fields charged under both E_8s of the uncompactified heterotic theory. We did not perform a systematic search through all modular invariant gauge shifts so we cannot make any conclusions about the existence of the Pyramid spectrum in these models, but we were unable to find the complete spectrum in our search and believe it likely that no gauge shift in the Z_3 will give rise to a vector-like set of trianons. If this is indeed the case, we are forced to abandon our hope that the Z_3 symmetry of the Pyramid model arises as an artifact of the geometry.

Despite our limited success, there still remain many avenues in the vast landscape of string models to explore. We are especially interested in continuing our search for the Pyramid model of TeV physics in the contexts of each of the three string theories we have investigated. We also believe that models of intersecting branes in type IIB theory and F-theory models might afford us the techniques required to build our desired low energy effective theory.

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References

[1] T. Banks, S. Echols and J. L. Jones, JHEP **0710**, 105 (2007) [arXiv:0708.0022 [hep-th]].

[2] T. Banks, "Cosmological supersymmetry breaking and the power of the pentagon: A model of low energy particle physics," arXiv:hep-ph/0510159.

[3] T. Banks, "Remodeling the pentagon after the events of 2/23/06," arXiv:hep-ph/0606313.

[4] K. A. Intriligator, N. Seiberg and D. Shih, "Dynamical SUSY breaking in meta-stable vacua," JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].

[5] J. L. Jones, "Gauge Coupling Unification in MSSM + 5 Flavors," arXiv:0812.2106 [hep-ph].

[6] T. Banks and H. haber, "Note on the pseudo-NG-boson of meta-stable SUSY breaking," in preparation.

[7] T. Banks and J. F. Fortin, "A Pyramid Scheme for Particle Physics," arXiv:0901.3578 [hep-ph].

[8] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, "Toward realistic intersecting D-brane models," Ann. Rev. Nucl. Part. Sci. **55**, 71 (2005) [arXiv:hep-th/0502005].

[9] M. Cvetic, G. Shiu and A. M. Uranga, "Chiral four-dimensional N = 1 supersymmetric type IIA orientifolds from intersecting D6-branes," Nucl. Phys. B **615**, 3 (2001) [arXiv:hep-th/0107166].

[10] M. Cvetic, I. Papadimitriou and G. Shiu, "Supersymmetric three family SU(5) grand unified models from type IIA orientifolds with intersecting D6-branes," Nucl. Phys. B **659**, 193 (2003) [Erratum-ibid. B **696**, 298 (2004)] [arXiv:hep-th/0212177].

[11] M. Cvetic, G. Shiu and A. M. Uranga, "Chiral type II orientifold constructions as M theory on G(2) holonomy spaces," arXiv:hep-th/0111179.

[12] M. Atiyah and E. Witten, "M-theory dynamics on a manifold of G(2) holonomy," Adv. Theor. Math. Phys. **6**, 1 (2003) [arXiv:hep-th/0107177].

[13] B. S. Acharya and E. Witten, "Chiral fermions from manifolds of G(2) holonomy," arXiv:hep-th/0109152.

[14] J. L. Bourjaily and S. Espahbodi, "Geometrically Engineerable Chiral Matter in M-Theory," arXiv:0804.1132 [hep-th].

[15] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, "Supersymmetric standard model from the heterotic string. II," Nucl. Phys. B **785**, 149 (2007) [arXiv:hep-th/0606187]. O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, "A mini-landscape of exact MSSM spectra in heterotic orbifolds," Phys. Lett. B **645**, 88 (2007) [arXiv:hep-th/0611095]. O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, "The Heterotic Road to the MSSM with R parity," Phys. Rev. D **77**, 046013 (2008) [arXiv:0708.2691 [hep-th]]. T. Kobayashi, S. Raby and R. J. Zhang, "Searching for realistic 4d string models with a Pati-Salam symmetry: Orbifold grand unified theories from heterotic string compactification on a Z(6) orbifold," Nucl. Phys. B **704**, 3 (2005) [arXiv:hep-ph/0409098]. S. Raby, "SUSY GUT model building," AIP Conf. Proc. **1078**, 128 (2009).

[16] Akin Wingerter. *Aspects of Grand Unification in Higher Dimensions*. 2005. PhD thesis BONNIR- 2005-05 ISSN-0172-8741. Patrick K. S. Vaudrevange. *Geometrical Aspects of Heterotic Orbifolds*. 2005. Diploma thesis BONN-IB-2005-08.

[17] H. P. Nilles, S. Ramos-Sanchez, P. K. S. Vaudrevange and A. Wingerter, "Exploring the SO(32) heterotic string," JHEP **0604**, 050 (2006) [arXiv:hep-th/0603086].

[18] R. Slansky, "Group Theory For Unified Model Building," Phys. Rept. **79**, 1 (1981).

[19] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono and K. Tanioka, "TABLES OF Z(N) ORBIFOLD MODELS,"

[20] J. E. Kim, "Trinification from superstring toward MSSM," arXiv:hep-ph/0310158. K. S. Choi and J. E. Kim, "Three family Z(3) orbifold trinification, MSSM and doublet-triplet splitting problem," Phys. Lett. B **567**, 87 (2003) [arXiv:hep-ph/0305002]. J. E. Kim, "Z(3) orbifold construction of SU(3)**3 GUT with sin**2(Theta(0)(W)) = 3/8," Phys. Lett. B **564**, 35 (2003) [arXiv:hep-th/0301177].

[21] K. S. Choi, K. Hwang and J. E. Kim, "Dynkin diagram strategy for orbifolding with Wilson lines," Nucl. Phys. B **662**, 476 (2003) [arXiv:hep-th/0304243].